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STRUCTURE NUMERICAL MODEL UPDATING ON THE RESULTS OF EXPERIMENTAL DYNAMIC TESTS

Abstract
The paper presents an algorithm of structure calculation model adjustment by the values of free oscillations frequencies of structures, obtained as a result of experimental research. It is considered that the experimental test of a construction can obtain some values of the natural frequencies of the object and identify the corresponding forms of free oscillations.

Keywords: computational model, natural frequencies, dynamic test, perturbation error

1. Introduction

Consider the problem of the adaptation of computational models of building structures during the dynamic monitoring or on the results of experimental studies of these structures. Suppose that in the experiment we succeeded in obtaining N natural frequencies of the real object $\hat{f}_j$ and in identifying the corresponding forms of free oscillations of the structure Φ_j:

$\hat{f}_i = \dfrac{\hat{\omega}_i}{2\pi}$ – i-th frequency of free oscillation of structure in a second, the resulting of field tests, measured in *hertz*,

$\hat{\omega}_i$ – corresponding i-th site cyclic (circular) oscillation frequency of structure, i.e. detected i-th natural frequency of the structure in 2π seconds.

Calculation model of buildings constructed for its numerical analysis in displacements, for example, based on the finite element method, in general, expressed in a matrix equation of movement:

$$[M]\cdot\{\ddot{u}\}+[C]\cdot\{\dot{u}\}+[K]\cdot\{u\}=\{P\} \qquad (1)$$

Thus, the construction calculation model is numerically represented by its matrix of stiffness $[K]$ and by the mass matrix $[M]$ (and also, in general case of the calculation, by the matrix of dissipative characteristics of structure $[C]$ and by the vector (vectors) of the external force influences on the structure $\{P\}$). Dimensions of square matrices $[K]$, $[M]$ and $[C]$ as well as dimension of vectors $\{P\}$ and of the unknown displacements vectors $\{u\}$ is n, where n – the number of degrees of freedom assigned to the structure numerical model.

The equation of motion of a discrete numerical model of a mechanical system without internal viscous friction has a matrix form:

$$[M]\cdot\{\ddot{u}\}+[K]\cdot\{u\}=\{P\}, \qquad (2)$$

and in the case of free movement

$$[M]\cdot\{\ddot{u}\}+[K]\cdot\{u\}=\{0\} \qquad (2a)$$

where $\{u\}$ – the displacements vector.

When taking the free vibration as harmonics, i.e. then $\{u\}=\{\Phi\}_i\cdot\cos(\omega_i\cdot t)$, the equations of free oscillations (2a) become

$$\left(-\omega_i^2\cdot[M]+[K]\right)\cdot\{\Phi\}_i=\{0\}, \qquad (3)$$
$$i = 1, 2,\ldots,n,$$

where:

ω_i – an arbitrary i-th site cyclic (circular) vibration frequency of the structure calculation model,

$\{\Phi\}_i$ – the form of free oscillations of the structure calculation model, corresponding to the its i-th natural frequency,

n – number of degrees of freedom of the calculation model of structure (the order of the stiffness matrix $[K]$ and mass matrix $[M]$).

2. Technique of adaptation of the computational model

Let's write the system of equations of stationary oscillations (3) in the full matrix form

$$[K]\cdot[\Phi]-[M]\cdot[\Phi]\cdot[J]=\{0\} \qquad (4)$$

where:

$[\Phi]=\left[\{\Phi\}_1\{\Phi\}_2\ldots\{\Phi\}_n\right]$ – the matrix whose columns are the eigenvectors of the calculation model of the structure.

If we take

$$[A]=[M]^{-1}[K] \qquad (5)$$

then the matrix form of the equations of free oscillations (4) can be written as

$$[A]\cdot[\Phi]=[\Phi]\cdot[J] \qquad (6)$$

$[J]=\operatorname{diag}(\lambda_1,\lambda_2,\ldots,\lambda_n)$, or Jordan form of the matrix [A],

$\lambda_1\le\lambda_2\le\ldots\le\lambda_n$ – eigenvalues of the matrix $[A]=[M]^{-1}[K]$, corresponding to the eigenvectors $\{\Phi\}_i$, and at the same time $\lambda_i=\omega_i^2$ – is the i-th eigenvalue of the structure calculation model, ω_i – the corresponding i-th site cyclic (circular) vibration frequency of the structure calculation model, i = 1, 2,…,n .

Adjustments to the structure computational model arbitrary using N values of frequencies of free oscillations $\hat{\lambda}_j=\hat{\omega}_j^{\,2}$, j = 1,…, N ($N\le n$) identified through field measurements are possible, if in the sequence of eigenvalues $\Lambda=\{\lambda_1,\lambda_2,\ldots,\lambda_n\}$ we substitute the values $\hat{\lambda}_j$ instead of the values λ_j corresponding to natural forms $\{\Phi\}_j$.

Thus, we get the changed sequence of eigenvalues $\hat{\Lambda}$, for example, of some kind $\hat{\Lambda}=\{\hat{\lambda}_1,\lambda_2,\ldots,\hat{\lambda}_j,\ldots,\hat{\lambda}_N,\ldots,\lambda_n\}$.

Minimizing average "loss" value with regard to the replacement of N eigenvalues λ_j of the calculation model for values $\hat{\lambda}_j$ identified from the experiment we carry out with the least squares method, if we at first construct, for example, a polynomial of the form

$$y(x)=\sum_{k=0}^{m}\alpha_k\cdot x^k \qquad (7)$$

smoothing the sequence $\hat{\Lambda}(i)$ from the condition

$$S=\sum_{i=1}^{n}\Delta_i^{\,2}\to\min$$

where $\Delta_i=y(x_i)-\hat{\Lambda}_i$, $x_i=i$.

Then by the coefficients α_k, k = 0, 1, 2,…, m of polynomial (7) obtained as a result we construct the corrected spectrum of eigenvalues $\{\tilde{\lambda}_1,\tilde{\lambda}_2,\ldots,\tilde{\lambda}_n\}$, calculating its elements by the formula (7):

$$\tilde{\lambda}_i=y(i) \qquad (8)$$

The computational model of the structure, adjusted according to the results of experimental studies, we represent by the matrix of the form (5):

$$[\tilde{A}]=[\Phi]\cdot[\tilde{J}]\cdot[\Phi]^{-1} \qquad (9)$$

formed according to the submission of its Jordan decomposition:

$$[\tilde{J}]=\operatorname{diag}(\tilde{\lambda}_1,\tilde{\lambda}_2,\ldots,\tilde{\lambda}_n) \qquad (10)$$

$$[\tilde{J}]=[\Phi]^{-1}\cdot[\tilde{A}]\cdot[\Phi] \qquad (11)$$

3. Maintaining the structure and the approximation error of the computational model

The matrix $[A]$ (9) adjustments made in such a way by changing the number of its eigenvalues retains the perturbation error of its Jordan form $[J]=\operatorname{diag}(\lambda_1,\lambda_2,\ldots,\lambda_n)$, but certainly violates its band structure. In turn, maintaining the matrix $[\tilde{A}]$ band structure becomes the essential perturbation to it. However, the conservation of the tape width of the matrix $[\tilde{A}]$ while preserving the original error of perturbation of the initial sequence of eigenvalues $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ of the calculation model is an important and, in general, is an integral component of the adjustment procedure of the complicated structure calculation model according to the results of dynamic tests.

To save the band structure of the adjusted matrix $[\tilde{A}]$ reduce the sensitivity of the system of equations (2) by its preconditioning with incomplete Cholesky decomposition of the symmetric, positive definite matrix $[A]$.

Choose a simple rarefied lower triangular matrix $[h]$, it being "sufficiently similar" to the matrix $[g]$ of the Cholesky decomposition of the matrix $[A]$:

$$[A] = \{g\} \cdot \{g\}^T \tag{12}$$

The matrix $[D]$, as an incomplete Cholesky decomposition of the matrix $[A]$ becomes the preconditioner of the system (2):

$$[D] = \{h\} \cdot \{h\}^T \tag{13}$$

The main criterion in appointing the preconditioner $[D]$ is the fulfillment of the condition:

$$cond(Ad) << cond(\tilde{A})$$

where $cond(Ad)$ – is the conditionality number of the matrix

$$[Ad] = [D]^{-1} \cdot [\tilde{A}], \tag{14}$$

as well $cond(\tilde{A})$ – is the conditionality number of the matrix $[\tilde{A}]$.

Further in a symmetric matrix $[Ad]$ (14) the elements will be taken into consideration only within the band width of the original matrix $[A]$. The rest of its elements will be taken as zero. We call such a matrix $[A\tilde{d}]$.

Thus, for example, in the case of the static analysis of structures the solution of the original equations system of the form

$$[A] \cdot \{u\} = \{P\} \tag{15}$$

is replaced with

$$[A\tilde{d}] \cdot \{u\} = [D]^{-1} \{P\}. \tag{16}$$

4. Conclusions

An approach is proposed that allows to adjust the structure computational model, using a number of values of natural frequencies obtained from experimental tests of the structure. The method allows to keep the perturbation error of the calculation model when changing the values of its natural frequencies, while maintaining the band structure of the stiffness matrix of the structure calculation model.

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