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Structure and Environment 2025, vol. 17, (2), pp. 75-82, Article number: el 008
https://doi.org/10.30540/sae-2025-008

Structure and Environment
ISSN 2081-1500
e-ISSN 2657-6902
https://content.sciendo.com/sae
https://sae.tu.kielce.pl

sciendo

Kielce University of Technology

DOI: 10.30540/sae-2025-008

APPLICATION OF CENTRAL COLLINEATION IN UNDERSTANDING OF INTERSECTION STRUCTURE OF SOLIDS IN THE ACTIVITY OF ENGINEER

ZASTOSOWANIE KOLINEACJI ŚRODKOWEJ W ANALIZIE PRZECIĘĆ BRYŁ W DZIAŁALNOŚCI INŻYNIERSKIEJ

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Abstract

Modern design takes place in systems that use solid geometry. There, you receive ready-made geometric models of complex architectural and construction elements. This is not always accompanied by the engineers full understanding of the spatial relationship between the elements creating a complex geometric object. The authors propose the use of central collineation in the creation and analysis of the correctness of spatial geometric structures.

Keywords: Intersection of solids, central (axial) collineation, Desargues' theorem, AutoCAD.

Streszczenie

Współczesne projektowanie odbywa się w systemach, gdzie wykorzystywana jest geometria brył. Otrzymuje się tam gotowe modele geometryczne złożonych elementów architektonicznych i budowlanych. Nie zawsze towarzyszy temu pełne zrozumienie przez inżynierów relacji przestrzennych między elementami tworzącymi złożony obiekt geometryczny. Autorzy proponują wykorzystanie kolineacji środkowej w nauczaniu tworzenia i analizy poprawności przestrzennych struktur geometrycznych.

Słowa kluczowe: przecięcie brył, centralna (osiowa) kolineacja, twierdzenie Desargues', AutoCAD.

1. INTRODUCTION

Design in construction and architecture, in its geometric part, is currently carried out in the CAD environment and is often implemented in 3D. What is important here is the structure of geometric elements and their mutual connections, as well as mapping in the classic convention of orthographic projections [1]. In geometric terms, it is a sequence of cross-sections and intersections of elements, their visualization and presentation in a orthographic projection, perspective or axonometry. In order to fully understand the geometric structure of the object constructed in 3D (and at the same time to check the correctness of its execution), it is advisable to perform a geometric analysis in rectangular and axonometric projections. This is particularly important in BIM technology,

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where individual interconnected parts of the project are mutually verified [2, 3]. In geometric verification, it is convenient to use a transformation called central (axial) collineation. There are four types of central (axial) collineation in the affine plane [4-6]. Individual implementations of alignment on the plane, depending on the affine nature of the center and axis of alignment, are illustrated in Figure 1. Further drawings illustrating the use of the transformation to analyze and create geometric structures are presented in the form of a cartoon (Figs. 2, 3, 5). It is important that flat realizations can also be interpreted as solutions to problems in 3D (Fig. 3).

The algorithmic nature of central collineation can become the basis for innovative approach to geometrical analysis and creation of structures both in RhinoGrasshopper and in Revit environments [7, 8]. The geometric analysis method supports the development of spatial imagination and understanding of 3D geometry in engineering.

2. AFFINE TYPES OF CENTRAL COLLINEATION

2.1. Affine plane

An *affine plane* is a set, whose elements are called points, and a set of subsets, called lines, satisfying the following three axioms, A1-A3.

A1. Given two distinct points P and Q, there is one and only one line containing both P and Q.

A2. Given a line l and a point P not on l, there is one and only one line m which is parallel to l and which passes through P.

A3. There exist three non-collinear points (A set of points $P_1, P_2, . . . , P_n$ is said to be collinear if there exists a line l containing them all.) [5].

2.2. Ideal points and the projective plane

Let Af be an affine plane. For each line l $l \in Af$, we will denote by $=\{l\}$ the pencil of lines parallel to l, and we will call $\{l\}$ an *ideal point*, or *point at infinity*, in the direction of l. We write $L^\infty=\{l\}$ [5].

We define the completion Pr of Af as follows. The points of Pr are the points of Af, plus all the ideal points of Af. A line in Pr is either:

a) an ordinary line l of Af, plus the ideal point $L^\infty=\{l\}$ of l,

or

b) the *line at infinity* l^∞, consisting of all the ideal points of Af.

As it turns out, the plane completed in this way Pr is the projective plane, in the sense of the following definition.

2.3. Projective plane

A *projective plane Pr* is a set, whose elements are called points, and a set of subsets, called lines, satisfying the following four axioms.

P1. Two distinct points P, Q of Pr lie on one and only one line.

P2. Any two lines meet in at least one point.

P3. There exist three non-collinear points.

P4. Every line contains at least three points [5].

2.4. Collineation and central collineation

A *collineation* (*homology*) is a one-to-one onto mapping of the projective plane to itself in which collinear points are mapped to collinear points. The collineation is *central* if there is a *center*, a point S (S^∞) where all lines on S (S^∞) are fixed, meaning the line is mapped to itself, although individual points on the line need not be fixed. A a collineation is central exactly when it has a line of fixed points, an *axis* [9].

Hence, central collineation can be considered as axial collineation. This property is justified by Desargues' theorem [8].

More precisely, the definition of central collineation is justified by the property of closing the Desargues configuration (DG) [10-13]. The essence of DG is that in creating the image of the triangle $A_1B_1C_1$, the lines A_1B_1, A_2B_2 (which „were not constructed") and the collineation axis turn out to be co-bundle (i.e. they pass through one point) (cf. [9, 13]).

Desargues' theorem forms the basis of the principles of perspective used in photography, art, and engineering.

We say that two triangles have a *perspective point*, when we can label them $A_1B_1C_1$ and $A_2B_2C_2$ in such a way, that lines A_1A_2, B_1B_2, C_1C_2 pass through a common point S (S^∞). Pairs of points A_1, A_2; B_1, B_2; C_1, C_2, are then called *homologous*. The point S (S^∞) is then called *perspectivity center* of the two triangles $A_1B_1C_1$ and $A_2B_2C_2$.

We say that two triangles have a *perspective line*, when we can label them $A_1B_1C_1$ and $A_2B_2C_2$, in such a way (see Fig. 1), that the points of intersection of pairs of lines $a_1(B_1C_1)$, $a_2(B_2C_2)$; $b_1(A_1C_1)$, $b_2(A_2C_2)$; $c_1(A_1B_1)$, $c_2(A_2B_2)$ are contained in the same line s (s^∞). Pairs of lines $a_1(B_1C_1)$, $a_2(B_2C_2)$; $b_1(A_1C_1)$, $b_2(A_2C_2)$; $c_1(A_1B_1)$, $c_2(A_2B_2)$ are then called *homologous*. Line s (s^∞) is called *perspectivity axis* of the two triangles $A_1B_1C_1$ and $A_2B_2C_2$.

Theorem. (*Desargues*) Two triangles have a perspectivity center if and only if they have a perspectivity axis [8].

An example of the construction of an axial affinity triangle image is shown in Figure 2.

structure

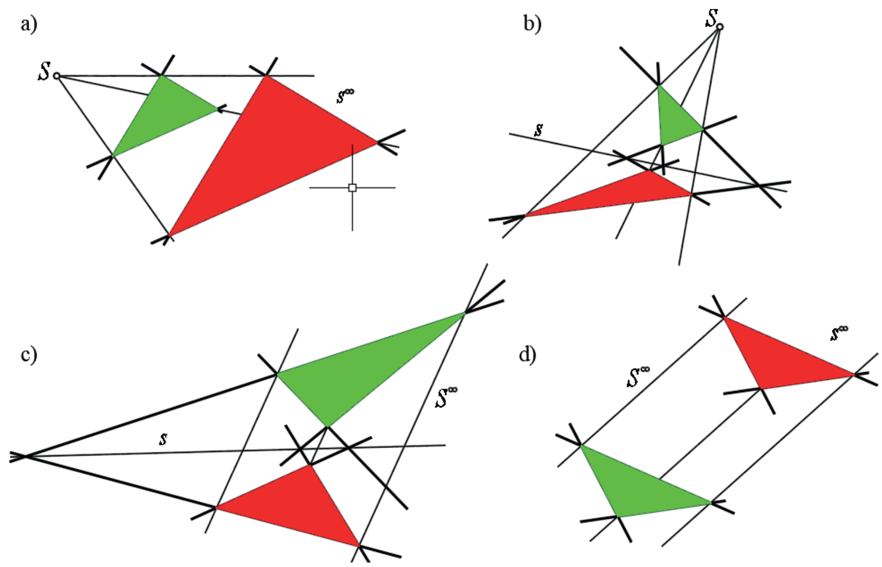

Fig. 1. Types of central collineation (homology) on a plane: a) central dilatation (center of collineation → ordinary point (S), axis of collineation → line at infinity (s^∞)), b) central collineation (center of collineation → ordinary point (S), axis of collineation → ordinary line (s)), c) axial affinity (center of collineation → ideal point (S^∞), axis of collineation → ordinary line (s)), d) translation (center of collineation → ideal point (S^∞), axis of collineation → line at infinity (s^∞))

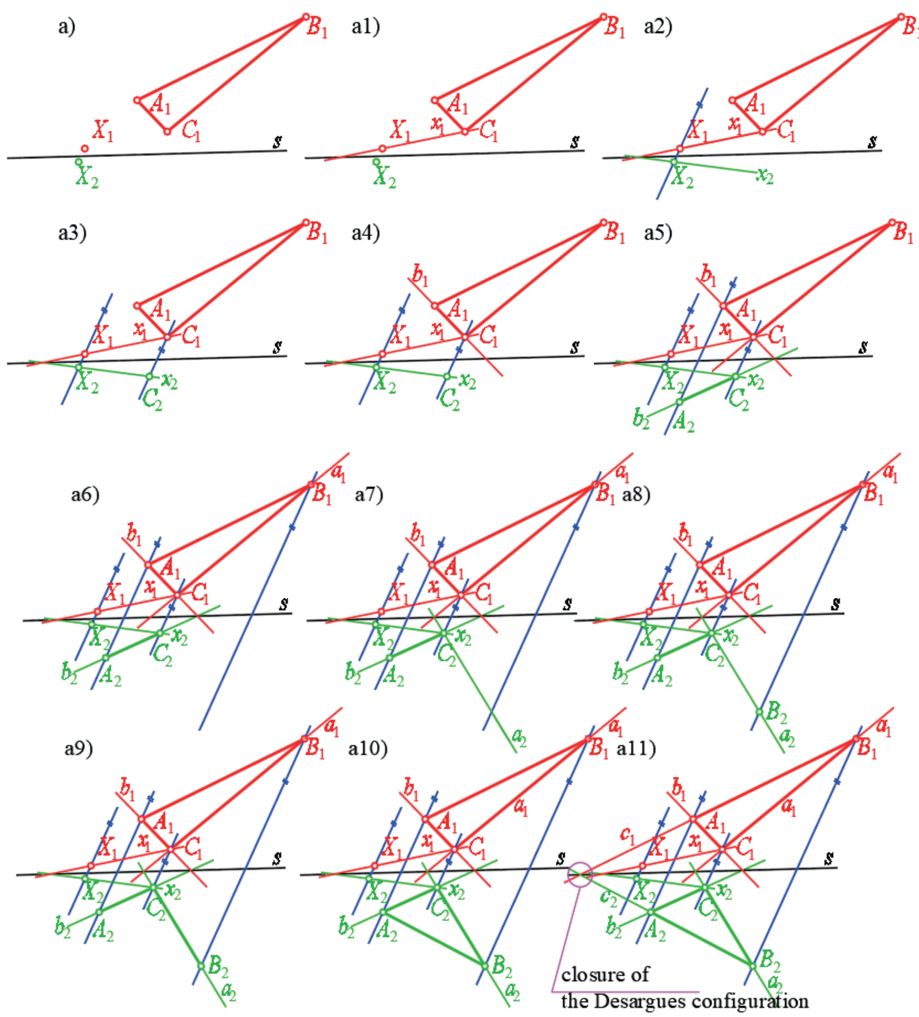

Fig. 2. The construction of the triangle $A_1B_1C_1$ image in axial affinity, closing of the Desargues configuration [9, 11, 12]

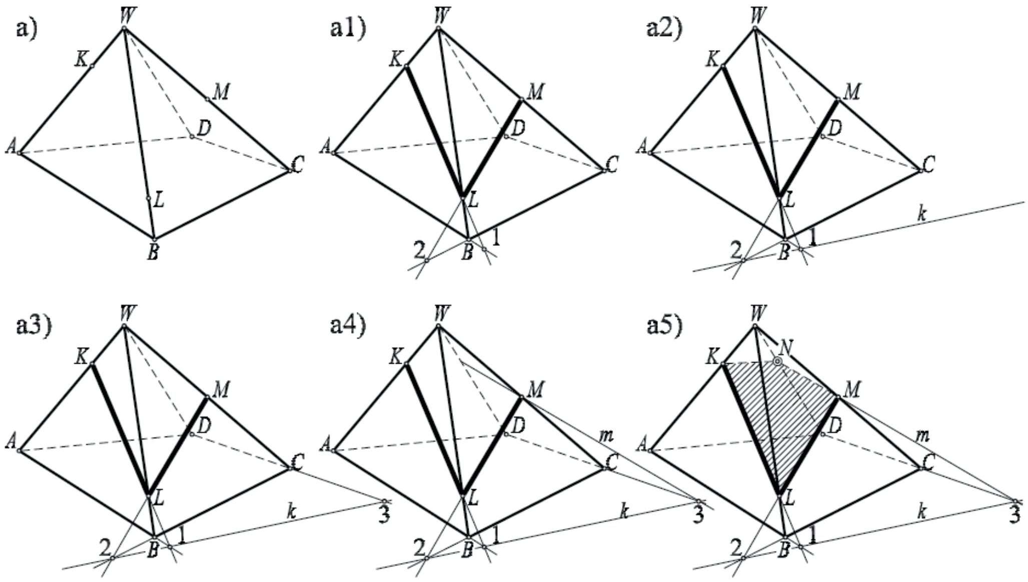

Fig. 3. Realization of axial (central) alignment in three-dimensional 3D space (axis of collineation k and center of collineation W) when finding a cross-section of a pyramid

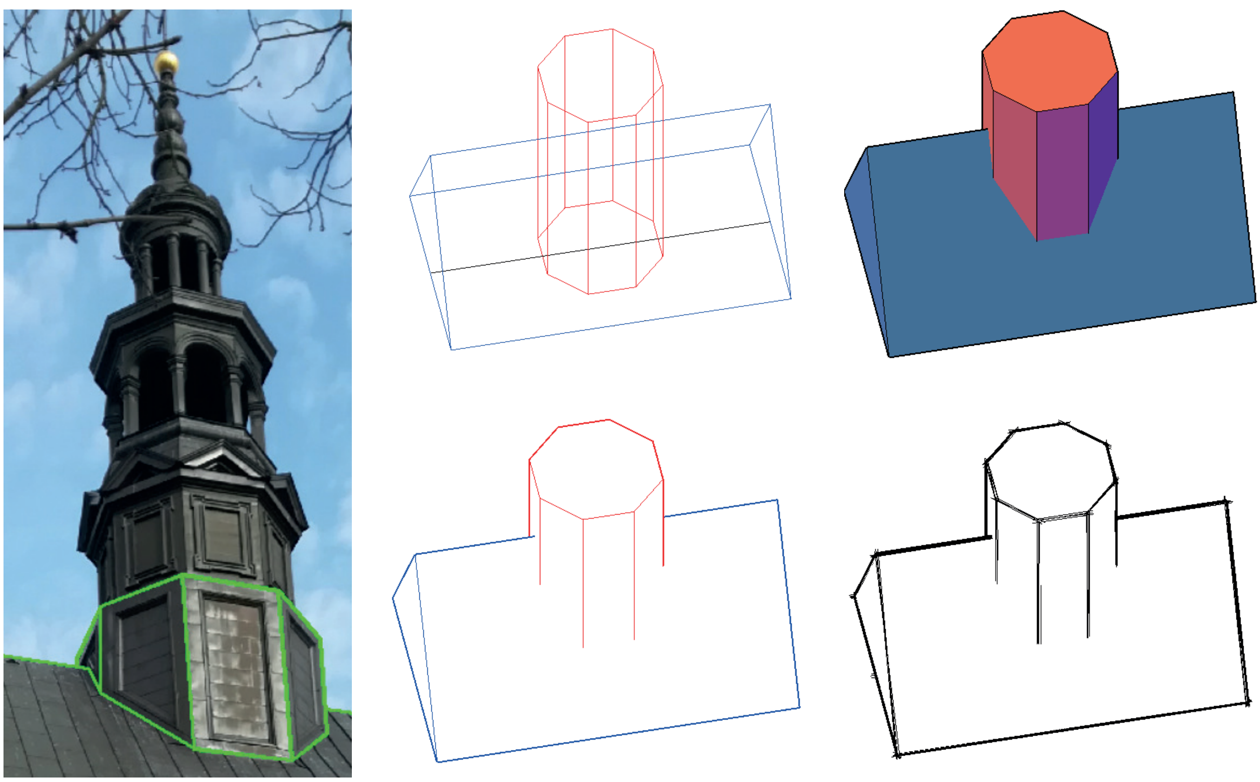

Fig. 4. Tower of the Cathedral Basilica of the Assumption of the Blessed Virgin Mary in Kielce. Interpenetration line of the tower with the roof (photo taken for the analysis of the problem 1). Next: a 3D model of the connection between the tower and the roof used for the geometric analysis of the structure Several AutoCAD visualization options: 3D wireframe, conceptual, 3D hidden, sketchy

structure

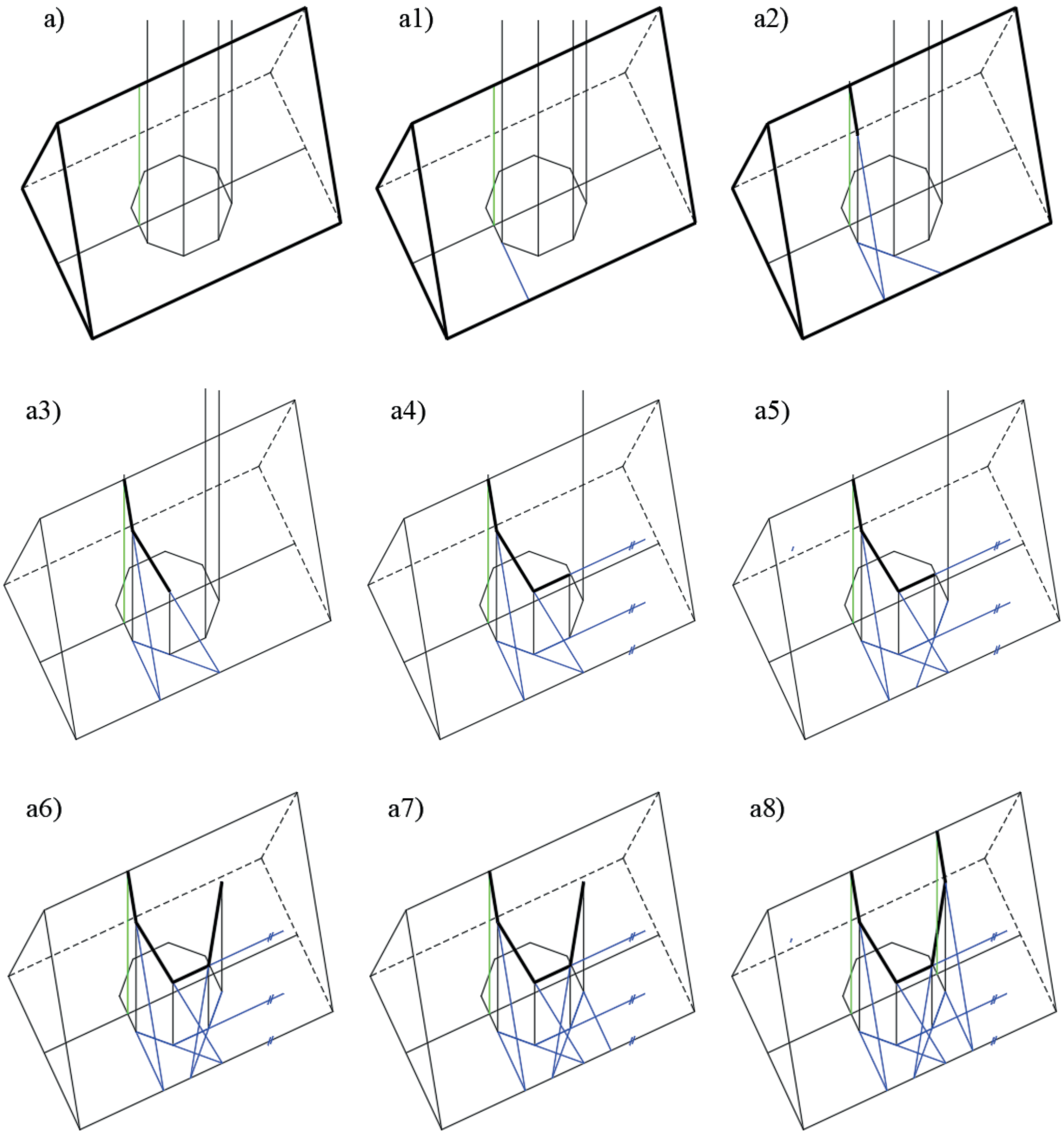

Fig. 5. Construction of the tower-roof interpenetration line using axial affinity

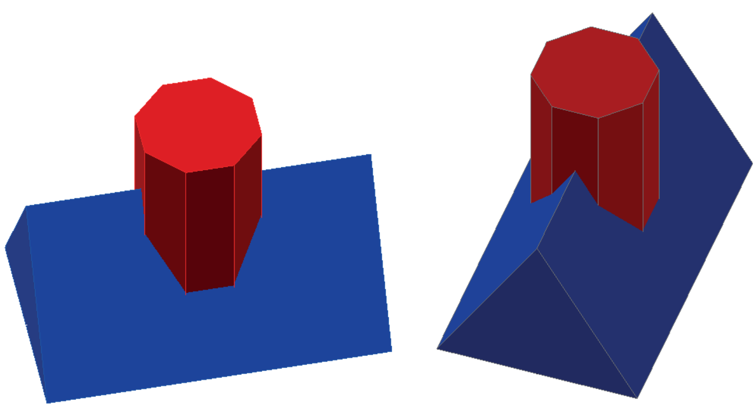

Fig. 6. The analyzed 3D model of the tower-roof interpenetration in realistic visualization in AutoCAD

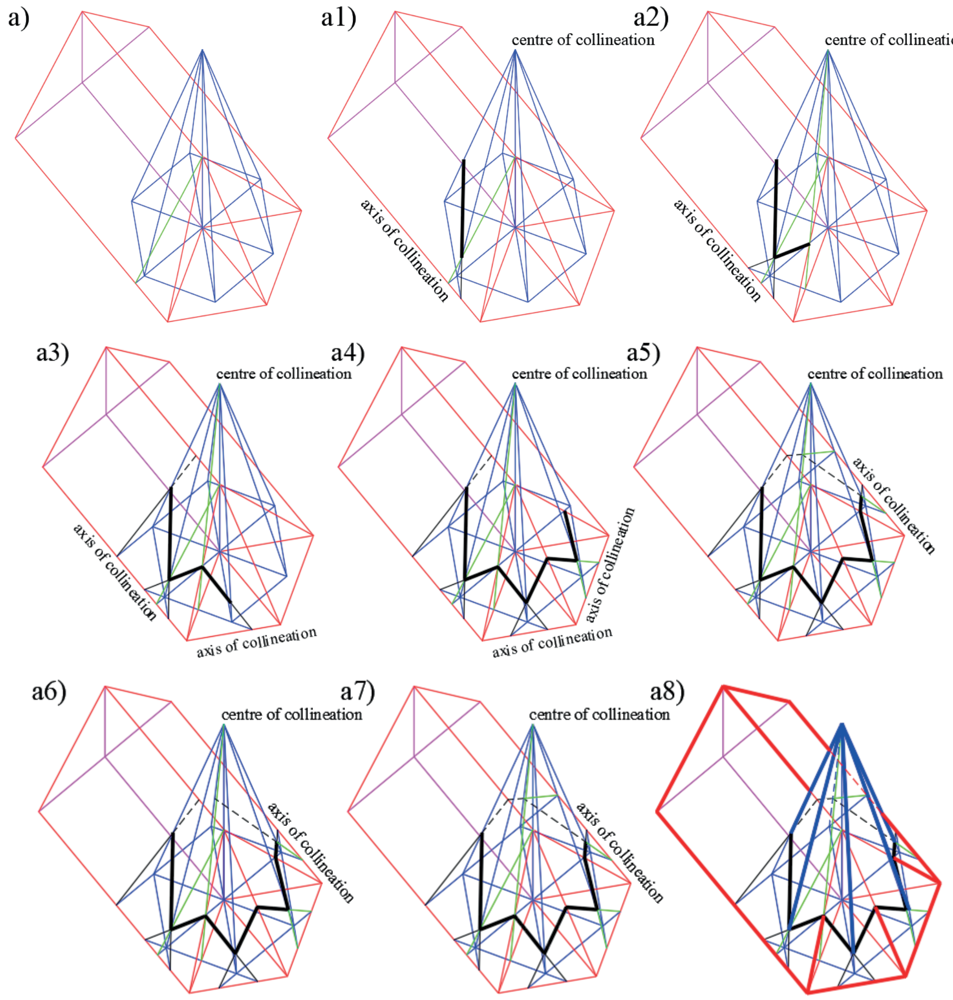

Fig. 7. Construction of the interpenetration line of a pyramid with a roof in axonometry – the method of collinear transformations

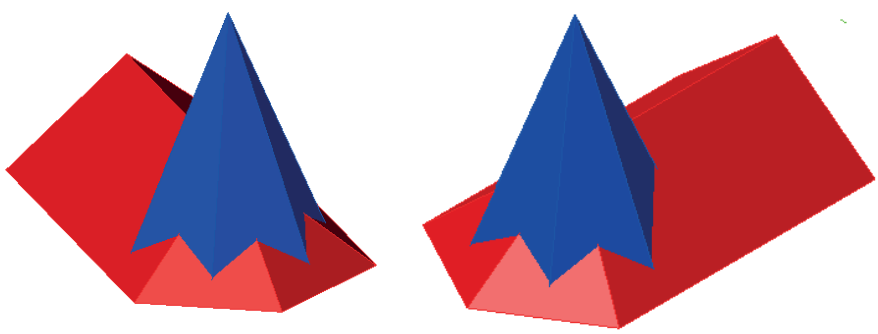

Fig. 8. 3D model of the interpenetration of a pyramid with a roof used to analyze the geometric structure

3. APPLICATION OF COLLINEATION – CASE STUDY

In this chapter, we present the use of collineation in three structures: cross-section (problem 1) and interpenetration of two solids (problems 2 and 3).

3.1. Problem 1

Task 1. Given: the pyramid *ABCDW* and the points *K*, *L*, *M* on its edges *AW*, *BW*, *CW* (Fig. 3a). Find the cross-section of the pyramid *ABCDW* with the α(*KLM*) plane.

To construct a cross-section of a given pyramid in projection, we proceed as shown in Figure 3. This is done by applying the so-called *nodal point theorem* (*For any three planes, the edges of each pair of these planes pass through one point* [14]) for several times. Just apply the procedure twice: find the common point of two edges and then the third edge must pass through this point. This is how we find points 1 and 2 defining the line *k*, which can be considered as the axis of collineation (Fig. 3a1, 3a2). Using this collineation, we find the remaining points of the section. The *KLMN* cross-section is an image of the quadrilateral *ABCD* in collineation with center *W* and axis *k*. Using the collineation with the *k* axis and the center *W*, we find the missing point of the cross-section (Fig. 3a3, 3a4). The *KLMN* cross-section is an image of the quadrilateral *ABCD* in the collineation with center *W* and axis *k* (Fig. 3a5).

3.2. Problem 2

How to justify (or verify) the shape of the intersection of the tower with the roof on the example of the Cathedral Basilica of the Assumption of the Blessed Virgin Mary in Kielce?

This is an example of the interpenetration of a gable roof (a prism with an isosceles triangle base) and a prism with a regular octagon base.

The following geometric task can be formulated on the basis of a photograph presented in Figure 4.

Task 2. Determine the lines of intersection of the gable roof with the tower, which geometrically is a prism with a regular octagon base. The angle of inclination of the gable roof to the horizontal projection is 60°. We solve the task in axonometry (Fig. 5).

To solve a 3D problem a planar construction has been developed in AutoCAD by using an axial affinity properties.

First, finding a point on the roof ridge in the axonometric projection and in the axonometric (pictorial) projection of the orthogonal projection (Fig. 5a, green line) as corresponding points, we determine the axial affinity, where the axis is the roof eaves (Fig. 5a1-5a2), and the direction is determined by the vertical edges of the tower. The points of the interference line are determined by applying the axial affinity (Fig. 5a3-5a8).

3.3. Problem 3

Central collineation can be also used in order to construct the penetration line between tower in the shape of a pyramid with a regular hexagonal base and a multi-slope roof in axonometry and in Monge projections (A). The solution using central collineation shows the arrangement of the interference lines.

Task 3. Determine the line of intersection of the multi-slope roof with the tower – a pyramid with a regular hexagon base in axonometry (Fig. 7a) and in horizontal projection (Fig. 9a).

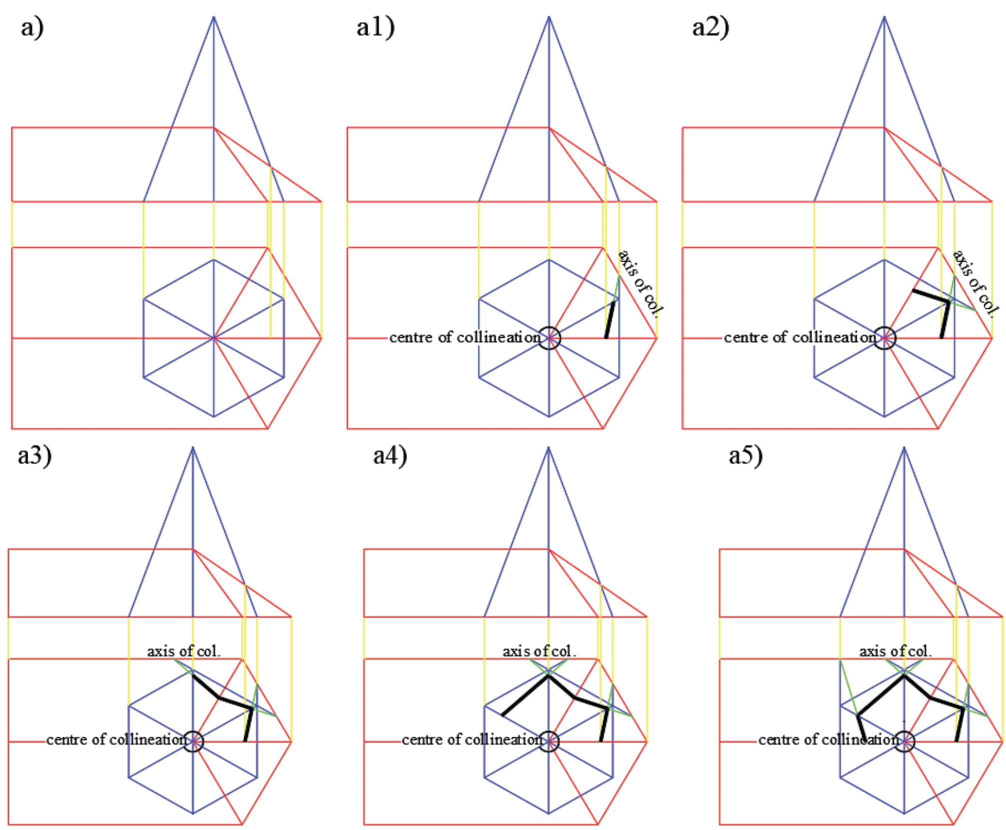

Fig. 9. Construction of the interpenetration line of a pyramid with a roof in Monge projections – the method of collinear transformations

First we solve the task in axonometry (Fig. 7a). In this case, we use four axial collineations with the same center and four different axes (Fig. 7a1-7a8).

We can compare the obtained flat drawing as a solution to task 3 with the one created in the AutoCAD environment 3D drawing (Fig. 8).

We can also use central collineation to create an object using orthographic (Monge) projections (Fig. 9). After determining the first point of the interference line (Fig. 9a), we construct a horizontal projection using two alignments with the same center (Fig. 9a1-9a5). Due to the symmetry of the horizontal projection in Figure 9, only part of it was made. Similarly, the construction of the vertical projection was omitted, which we supplement using the standard method of common elements.

4. CONCLUSION

The first quarter of the 21st century is coming to an end, and drawing is still one of the basic elements of everyday work for every engineer. Drawing is an inseparable element both in the design office and on the construction site, because it is impossible to imagine even the simplest project without presenting it in the form of a drawing. Drawings must be made according to strictly defined rules so that there is always a possibility of determining the real shape and dimensions of the designed structure and the correct location in three-dimensional space. From a geometric point of view, making such drawings is simply a certain transformation of a spatial or flat figure into a flat figure, which falls within the field of projective mappings

As it has been shown in the solved here tasks, central collineation can be successfully used when creating three-dimensional geometric objects in engineering practice. The solution method is then orderly, clear and shows the spatial system of the structure. The presented method (cartoon film) is sparse in text and reading the drawings is enough to understand the structure. The method presented fits into the general trend in interpersonal communication of moving away from text in favor of images in the form of drawings, graphics, diagrams, photographs, icons, emoticons, etc. This is especially important in teaching, in the education of engineers, in textbooks and various other teaching aids. This is the role of this article.

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