



INFLUENCE OF RANDOM VARIABLE CORRELATION ON THE RELIABILITY OF STEEL STRUCTURES

WPŁYW KORELACJI ZMIENNYCH LOSOWYCH NA NIEZAWODNOŚĆ KONSTRUKCJI STALOWYCH

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Abstract

The study investigated the influence of correlations between random variables on the reliability index of selected structural systems. The structural design parameters were defined as deterministic quantities and random variables, and the correlation between them was modeled using four correlation-matrix variants. The ultimate limit state was adopted as the safety criterion, and appropriate limit state functions were formulated to identify structural failure. A sensitivity analysis of the reliability index with respect to the random variables was carried out using the Hasofer-Lind reliability index as the performance measure. The First-Order Reliability Method served as the primary computational approach, with Monte Carlo simulation as the reference method. All calculations were performed using the NUMPRESS Explore. The analyses demonstrated that increasing the correlation coefficient leads to higher values of the reliability index. The results confirm that the correlation coefficient significantly affects the reliability assessment and should not be neglected in structural safety evaluations.

Keywords: correlation coefficient, FORM method, Monte Carlo method, random variables, reliability index, sensitivity analysis

Streszczenie

W pracy analizowano wpływ korelacji zmiennych losowych na wartość wskaźnika niezawodności wybranych układów konstrukcyjnych. Parametry projektowe konstrukcji zdefiniowano jako wielkości deterministyczne oraz zmienne losowe. W analizie uwzględniono korelację zmiennych losowych poprzez zastosowanie czterech wariantów macierzy korelacji. Stan graniczny nośności przyjęto jako kryterium oceny bezpieczeństwa konstrukcji, a do identyfikacji stanu awarii zastosowano odpowiednie funkcje graniczne. Przeprowadzono analizę wrażliwości wskaźnika niezawodności względem zmiennych losowych, przyjmując wskaźnik Hasofera-Linda jako miarę niezawodności. Podstawową metodą obliczeniową była metoda niezawodności pierwszego rzędu (FORM), natomiast metodę Monte Carlo wykorzystano jako metodę referencyjną. Obliczenia przeprowadzono z wykorzystaniem oprogramowania NUMPRESS Explore. Na podstawie przeprowadzonych analiz zaobserwowano, że wzrost wartości współczynnika korelacji prowadzi do zwiększenia wartości wskaźnika niezawodności. Uzyskane wyniki potwierdziły, że współczynnik korelacji wywiera istotny wpływ na wartość wskaźnika niezawodności, wobec czego nie powinien być pomijany w ocenie bezpieczeństwa konstrukcji.

Słowa kluczowe: współczynnik korelacji, metoda FORM, metoda Monte Carlo, zmienne losowe, wskaźnik niezawodności, analiza wrażliwości

1. INTRODUCTION

A primary concern in the design of engineering structures is ensuring an adequate level of safety throughout the intended service life. The concept

of safety is inherently linked to assessing structural reliability. Reliability assessment of structures is a fundamental aspect of engineering analysis, aimed at quantifying the probability of safe performance

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under variable loading conditions. Approximate methods such as the First-Order Reliability Method (FORM) and the Second-Order Reliability Method (SORM) are commonly employed. FORM linearly approximates the limit state surface, enabling rapid calculation of the reliability index β . In contrast, SORM accounts for the curvature of this surface, thereby improving accuracy for nonlinear limit-state functions. Simulation-based approaches, including Monte Carlo methods and variance-reducing variants, allow precise estimation of the probability of failure, albeit at a higher computational cost. The assessment relies on a probabilistic description of the random variables representing loads and material strength, considering both their distributions and mutual dependencies. Correlation between variables can significantly affect failure probabilities; therefore, its proper modeling – using classical correlation coefficients or copula-based approaches – is a crucial element of reliability analysis. Comprehensive discussions and theoretical foundations of structural reliability methods used in civil engineering can be found in classical works [1-6].

In recent years, accounting for correlations among random variables in structural reliability analysis has become increasingly important. Classic assumptions of independence among random parameters – such as loads, material strengths, or material properties – can lead to underestimating or overestimating failure risk. In practice, many variables are interdependent, and their relationships significantly influence the probability of exceeding the limit-state function. Early foundational work by Hasofer and Lind [7] and later by Ditlevsen and Madsen [8] emphasized that the reliability index and failure probability can be significantly affected when dependencies among loads, material properties, and geometrical parameters are ignored. Subsequent studies expanded on these findings by demonstrating that independence assumptions often lead to misestimating reliability – either overly conservative or unconservative – depending on the nature and magnitude of correlations.

Classical correlation coefficients, such as Pearson's or Spearman's, measure linear or monotonic relationships but are limited for non-normal or nonlinear variables. In particular, copula-based approaches [9-11] have gained prominence for their flexibility in capturing complex, nonlinear dependence patterns. A copula is a function that separates the marginal distributions of individual variables (i.e., the behaviour of each variable on its own) from their dependence structure (i.e., how the variables are interdependent), allowing

realistic modelling of nonlinear, asymmetric, or tail-dependent correlations. These methods provide more accurate joint distributions in structural systems, especially when variables follow different probability distributions, and are widely applied in structural reliability analysis.

In parallel, reliability methods such as FORM and SORM [12] and simulation-based techniques (Monte Carlo, Importance Sampling) [13] have been adapted to handle correlated inputs. Work by Der Kiureghian [14] has shown that correlation effects can substantially change the geometry of the limit-state function, highlighting the importance of accurate modeling for both component and system reliability evaluation.

Modern approaches to modeling dependencies among variables include the use of copulas, which capture both linear and nonlinear correlations. Bai et al. [15] proposed a method that accounts for multiple correlation features, including dependencies among stress and strength, failure mechanisms, and structural components. A time-varying hybrid copula was employed to realistically model evolving dependency structures, and the results showed that including correlations yields more conservative and safer reliability estimates.

Similarly, Zhang and Yan [16] analyzed parallel-series systems in a stress-strength model, assuming a correlation between stress and strength using a Clayton copula. They applied a two-step maximum-likelihood estimation and the maximum-product-spacing method to estimate parameters, and conducted Monte Carlo simulations to assess the effectiveness of the estimation methods. The results indicated that ignoring dependencies among variables can lead to significant errors in determining system reliability.

In engineering applications, Kubicka and Radoń [17] focused on the system reliability of steel trusses, accounting for correlations among the strengths of individual members. FORM (First-Order Reliability Method) and Subset Simulation were employed, along with a sensitivity analysis to identify which components most influence overall structural reliability. Their study showed that increasing correlation among member strengths decreases the reliability index, thereby increasing the risk of structural failure.

Additional methodological contexts include reports from IFTR PAN and doctoral research [18] on third-level reliability methods that fully account for the joint distribution of random variables, including correlations. The literature also highlights that correlations may change over time due to aging or

material fatigue, necessitating dynamic methods, such as time-varying copulas.

In summary, the literature analysis shows that accounting for correlations among random variables is crucial for a realistic assessment of structural reliability. Copulas and probabilistic simulation methods, such as Monte Carlo or Subset Simulation, enable accurate risk evaluation, while sensitivity analysis helps identify which correlations have the most significant impact on structural safety. Contemporary studies confirm that ignoring dependencies can lead to significant errors in design and reliability assessment, whereas accounting for them enhances the credibility of analyses and improves the safety of engineering projects.

In the present study, the primary method employed was the First-Order Reliability Method (FORM). In contrast, the Monte Carlo method, recognized as the most accurate approach in structural safety assessment, was used as a reference to verify the correctness of FORM-based calculations. The Hasofer-Lind reliability index β served as the measure of structural reliability. A significant advantage of the FORM method is its ability to compute the sensitivity of the reliability index with respect to the random variables. This sensitivity, denoted by α , is calculated as the first derivative of the reliability index β with respect to the specified variable.

2. RESEARCH ASSUMPTIONS AND METHODS

2.1. Correlation between random variables

It is assumed in this paper that only one failure mode is considered and that a reliability measure for this failure mode is to be estimated. Further, it is believed that this failure mode can be mathematically formulated. An important step in a reliability analysis is to decide which quantities should be modelled as probabilistic variables and which as deterministic parameters. The probabilistic variables are denoted $\mathbf{X} = \{X_1, X_2, \dots, X_n\}$. Then probability could model physical uncertainty, model uncertainty or statistical uncertainty. The physical probabilistic can be load variables (e.g., traffic load), resistance variables (e.g., yield strength) or geometrical variables (e.g., length or cross-sectional area of a beam). The variables in \mathbf{X} are also denoted basic variables. Realizations of the basic variables are denoted $x = \{x_1, x_2, \dots, x_n\}$, i.e., x is a point in the n -dimensional basic variable space.

The joint density function for the probabilistic variables X is denoted $f_X(x)$. The elements of the vector of expected values and the covariance vector are:

$$\mu_i = E[X_i], \quad i = 1, \dots, n \quad (1)$$

$$C_{ij} = Cov[X_i, X_j], \quad i, j = 1, \dots, n \quad (2)$$

The standard deviation of X_i is denoted σ_i . The variance of X_i is $\sigma_i^2 = C_{ii}$. The coefficient of correlation between X_i and X_j is defined as:

$$\rho_{ij} = \frac{C_{ij}}{\sigma_i \sigma_j}, \quad i, j = 1, \dots, n \quad (3)$$

It is easy to see that $-1 \leq \rho_{ij} \leq 1$.

A complicated part of reliability analysis with correlated random variables (from the \mathbf{X} vector) is calculating the covariance vector: covariance vector:

$$C_{ij} = Cov[X_i, X_j] = E[(X_i - \mu_{X_i})(X_j - \mu_{X_j})] = E[X_i X_j - X_i \mu_{X_j} - \mu_{X_i} X_j + \mu_{X_i} \mu_{X_j}] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x_i - \mu_{X_i})(x_j - \mu_{X_j}) f_{X_i, X_j}(x_i, x_j) dx_i dx_j \quad (4)$$

For the n -dimensional variable, we have n variances and $n(n-1)$ covariance. We present this collection in the form of a covariance matrix:

$$C = \begin{bmatrix} C_{ii} & C_{ij} & \dots & C_{in} \\ C_{ji} & C_{jj} & \dots & C_{jn} \\ \vdots & \vdots & \ddots & \vdots \\ C_{ni} & C_{nj} & \dots & C_{nn} \end{bmatrix} \quad i, j = 1, \dots, n, i, j = 1, \dots, n \quad (5)$$

In analogy to the covariance matrix, a symmetrical correlation matrix is defined:

$$\rho = \begin{bmatrix} \rho_{ii} & \rho_{ij} & \dots & \rho_{in} \\ \rho_{ji} & \rho_{jj} & \dots & \rho_{jn} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{ni} & \rho_{nj} & \dots & \rho_{nn} \end{bmatrix} \quad i, j = 1, \dots, n, i, j = 1, \dots, n \quad (6)$$

Knowledge of the distribution parameters of the random variables (mean, standard deviation, and probability density function) does not allow direct estimation of their correlation coefficient. Therefore, the primary objective was not to precisely determine the correlation coefficient for individual variable pairs, but rather to demonstrate that the values of the correlation coefficient influence overall structural safety. In the reliability analysis, well-established methods for assessing the probability of failure, such as FORM and Monte Carlo simulations, were used.

2.2. Reliability software

In the examples presented below, the NUMPRESS Explore software [19] was employed for the reliability analysis. The NUMPRESS is a computational system developed at the Institute of Fundamental

Technological Research, Polish Academy of Sciences (IFTR PAN), dedicated to the probabilistic reliability analysis of engineering processes.

The core of software is its probabilistic module, which allows users to define parameters of random variables: mean, standard deviation, and probability density function. The next step is to define the limit-state functions representing critical conditions in standard mathematical notation, expressed as functions of the basic random variables. A correlation matrix, which represents relationships among random variables, can also be defined. The next stage involves selecting the method of reliability analysis and starting the computations. The basic information generated by the reliability program is the reliability index β and its corresponding failure-probability level.

Reliability software also offers advanced post-processing capabilities, including visualization of probability distributions and sensitivity analyses. These tools help identify critical parameters, evaluate their influence on system performance, and support the design of processes resistant to uncertainty. By integrating reliability analysis with numerical simulations, NUMPRESS provides an effective environment for realistic risk assessment. It enables engineers and researchers to improve the safety, stability, and predictability of engineering processes affected by variability.

3. RESULTS

In this study, the reliability of three types of steel structures was assessed, and the Hasofer-Lind reliability index β was estimated. The sensitivity of the reliability index to the considered random variables was also evaluated. Correlation matrices were defined based on engineering judgment and available statistical data. Correlations were assumed among variables that are physically or functionally related, such as material properties, cross-sectional dimensions, and loads, because they significantly affect structural behavior and the reliability index. Less related variables were treated as independent to simplify the model without compromising accuracy. The effect of the assumed correlation coefficients was examined through four cases of the correlation matrix. For each structure, four scenarios representing different relationships among the random variables were analyzed:

Case A – uncorrelated $\rho = 0$,

Case B – weak correlation $\rho = 0.3$,

Case C – medium correlation $\rho = 0.5$,

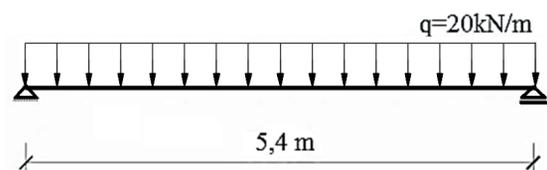
Case D – strong correlation $\rho = 0.9$.

A linear-elastic material model and the assumption of small displacements were adopted in the analysis. All random variables were assumed to follow a normal distribution. The coefficients of variation were selected based on statistical studies of building strength, construction materials, and products [20]. For variables related to loads, the coefficients of variation were chosen in accordance with the recommendations of the Joint Committee on Structural Safety [21]. For each structure, the following parameters were defined for the random variables considered in the reliability analysis (Table 1-Table 3): the random variable (X_i), its probability density function (p_j), the mean value (μ_x), the standard deviation (σ_x) and the coefficient of variation (v_x). These parameters provide a complete statistical characterization of the uncertainties associated with each variable and form the basis for subsequent probabilistic and sensitivity analyses.

3.1. Example 1 - steel beam

In Example 1, the reliability analysis was performed for a steel I-beam IPE270. The elements were made of S235 steel with a yield point $f_y = 235$ MPa and a beam section modulus $W_{pl,y} = 4.84 \cdot 10^{-4}$ m³. The structural element is a simply supported beam under a uniform load $q = 20$ kN/m (Fig. 1). A detailed numerical analysis of the reliability problem of this element has been presented earlier [22], but it did not include correlations among random variables. The details of the steel beam are shown in Figure 1.

a)



b)

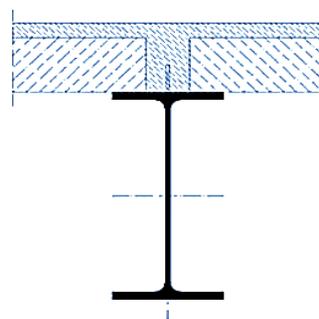


Fig. 1. Steel beam geometry, showing (a) structural scheme and (b) cross-sectional details [22]

The structural design parameters were treated as random variables. The probabilistic quantities incorporated into the analysis include: X_1 – uniform load q , X_2 – beam length L , X_3 – plastic section modulus $W_{pl,y}$, and X_4 – yield strength of S235 steel f_y . A detailed specification of these random variables is presented in Table 1.

Table 1. Characteristics of the random variables for the steel beam

X_i	p_i	μ_x	σ_x	v_x
X_1	Normal	20 kN/m	2 kNm	10%
X_2	Normal	5.4 m	0.0054 m	0.1%
X_3	Normal	$4.84 \cdot 10^{-4} \text{ m}^3$	$0.242 \cdot 10^{-4} \text{ m}^3$	5%
X_4	Normal	$235 \cdot 10^3 \text{ kN/m}^2$	$18.80 \cdot 10^3 \text{ kN/m}^2$	8%

The limit state function, representing the condition that the section’s bearing capacity is not exceeded under bending moment, was formulated. The maximum bending moment, expressed as a function of the random variable vector $\mathbf{X} = \{X_1, X_2, X_3, X_4\}$, was then determined using the following expression:

$$M_{Ed} = \frac{X_1 X_2^2}{8} \quad (7)$$

Limit function connected with ultimate limit state was formulated as follows:

$$G_1(\mathbf{X}) = 1 - \frac{M_{Ed}}{M_{c,Rd}} \quad (8)$$

where: for the first class elements

$$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl,y} f_y}{\gamma_{M0}}; \gamma_{M0} = 1.$$

By substituting all relevant parameters and representing the function in terms of the random variable vector $\mathbf{X} = \{X_1, X_2, X_3\}$, the limit state function is expressed as follows:

$$G_1(\mathbf{X}) = 1 - \frac{X_1 X_2^2}{8 X_3 X_4} \quad (9)$$

The Hasofer-Lind reliability index was determined using the FORM method, and the Monte Carlo method was used solely to verify the accuracy of the calculations. The reliability index values for the simply supported beam, corresponding to different correlation-matrix cases, are presented in Table 2.

Table 2. Calculated values of the reliability index (β)

Probabilistic method	Case of correlation matrix			
	Case A ($\rho = 0$)	Case B ($\rho = 0.3$)	Case C ($\rho = 0.5$)	Case D ($\rho = 0.9$)
FORM	3.86	4.28	5.29	7.89
Monte Carlo	3.88	4.31	5.30	7.90

3.2. Example 2 – steel arch with hangers and tie beam

In Example 2, the reliability of a steel arch with hangers and a tie beam under a uniformly distributed load was analyzed (Fig. 2). The structure was designed using I-beam 200 sections, and all elements were made of S235H steel with a yield point of $f_y = 215 \text{ MPa}$.

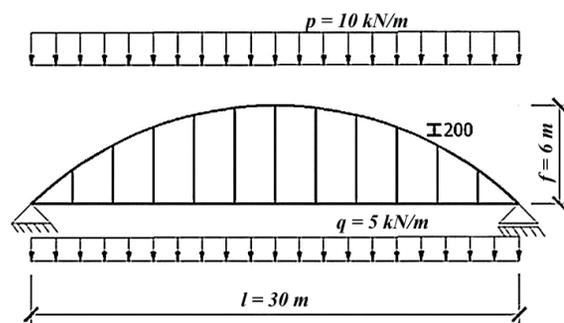


Fig. 2. Schematic of the steel arch with hangers and tie beam

The structure was assumed to be restrained against torsional instability perpendicular to the plane in which it is located, and the ultimate limit state function was defined based on the maximum compressive force. The cross-sectional area of the elements (A), the span (l), the arch rise (f), and the angle (ψ), defined as the angle between the tangent to the arch and the horizontal, were considered deterministic parameters. In the reliability analysis, the random variables considered were X_1 , representing the permanent load (p); X_2 , representing the variable load (q); and X_3 , representing the yield point for S235H steel (f_y). A detailed description of these random variables is provided in Table 3.

Table 3. Description of the random variables for steel arch

X_i	p_i	μ_x	σ_x	v_x
X_1	Normal	10 kN/m	0.6 kN/m	6%
X_2	Normal	5 kN/m	1 kN/m	20%
X_3	Normal	$235 \cdot 10^3 \text{ kN/m}^2$	$18.80 \cdot 10^3 \text{ kN/m}^2$	8%

The strength of the tie beam is expressed by the following relation:

$$H = \frac{(X_1 + X_2)l^2}{8f} \quad (10)$$

where: l indicates the beam span ($l = 30$ m) and f denotes the arch's rise ($f = 6$ m).

The maximum compressive force, corresponding to the axial force at the support, is then determined using the expression:

$$N_{ed} = \frac{H}{\cos\psi} = \frac{(X_1 + X_2)l^2}{8f \cos\psi} \quad (11)$$

The limit state function for the ultimate limit state, used in the reliability assessment, was formulated as follows:

$$G_2(\mathbf{X}) = 1 - \frac{|N_{ed}|}{N_{b,Rd}} \quad (12)$$

where: $N_{b,Rd}$ – I-beam capacity due to buckling is given by the formula $N_{b,Rd} = \chi \cdot A \cdot X_3 / \gamma_{M0}$ ($A = 0.00335$ m², $\gamma_{M0} = 1$), χ – buckling factor ($\chi = 0,315$).

By substituting all relevant parameters and representing the function in terms of the random variable vector $\mathbf{X} = \{X_1, X_2, X_3\}$, the limit state function is expressed as follows:

$$G_2(\mathbf{X}) = 1 - \frac{900(X_1 + X_2)}{0.0396 X_3} \quad (13)$$

The Hasofer–Lind reliability index was determined using the FORM method, and the Monte Carlo method was used solely to verify the accuracy of the calculations. The values of the reliability index for the steel arch under different correlation matrix cases are presented in Table 4.

Table 4. Calculated values of the reliability index (β)

Probabilistic method	Case of correlation matrix			
	Case A ($\rho = 0$)	Case B ($\rho = 0.3$)	Case C ($\rho = 0.5$)	Case D ($\rho = 0.9$)
FORM	3.49	3.84	4.63	7.36
Monte Carlo	3.43	3.72	4.58	7.34

3.3. Example 3 – steel industrial building

In Example 3, the reliability of a steel industrial building was analyzed. The structure consists of steel frames with rigid ridge, column-to-foundation, and beam-to-column connections (Fig. 3). Elements are made of S235JR steel $f_y = 235$ MPa, $E = 210$ GPa,

with columns of HEB 450 and girts of HEA 700 I-shaped beams. A detailed numerical analysis of the reliability problem of this element has been presented earlier [23]. Previous analyses did not account for correlations between random variables.

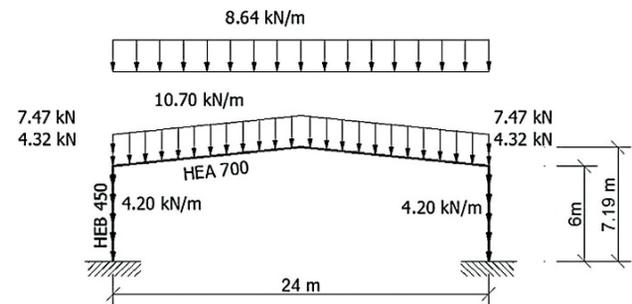


Fig. 3. Supporting structure of the steel industrial building with the most critical load combination [23]

Member section areas, member inertia moments, and column and beam lengths were assumed to be deterministic quantities. In the reliability analysis the following random variables were considered – roof self-weight per meter, – curtain wall self-weight per meter, – roof and wall fragment at the eave as a concentrated column load, – uniform snow load on the roof, – eave fragment as a concentrated column load, – modulus of elasticity of S235JR steel, and – yield strength of S235JR steel. The characteristics of the random variables are summarized in Table 5.

Table 5. Characteristics of the random variables for the steel industrial building

X_i	p_i	μ_x	σ_x	ν_x
X_1	Normal	10.70 kN/m	0.642 kN/m	6%
X_2	Normal	4.20 kN/m	0.252 kN/m	6%
X_3	Normal	7.47 kN	0.4482 kN	6%
X_4	Normal	8.64 kN/m	1.728 kN/m	20%
X_5	Normal	4.32 kN	0.864 kN	20%
X_6	Normal	$210 \cdot 10^6$ kN/m ²	$10.5 \cdot 10^6$ kN/m ²	5%
X_7	Normal	$235 \cdot 10^3$ kN/m ²	$18.80 \cdot 10^3$ kN/m ²	8%

The first stage involved a static analysis performed using Robot Structural Analysis Professional, which enabled the identification of the location of the maximum bending moment for the most critical load combination (Fig. 3). A preliminary analysis showed that the highest internal forces in the cross-sections

of columns and beams were obtained for the load combination of permanent load Q and snow S , defined as KOMB1: $1.35 \cdot 0.85 \cdot Q + 1.5 \cdot S$. Other combinations, including wind effects, produced similar distributions of internal forces (with no change in the sign of the bending moments), but with lower magnitudes. Therefore, in the subsequent calculations, wind effects are neglected. The next step was a symbolic analysis conducted in Mathematica. By incorporating both deterministic values and random variables into the finite element method algorithm, a formula for the maximum bending moment as a function of random variables grouped in vector: $\mathbf{X} = \{X_1, X_2, X_3, X_4, X_5, X_6, X_7\}$.

The limit state function associated with the ultimate limit state is formulated as follows:

$$G_3(\mathbf{X}) = 1 - \frac{\left| \frac{M_{max}}{W_s} \right|}{f_y} \tag{14}$$

where: f_y – yield point for S235JR steel ($f_y = X_7$), W_s – denotes the elastic index of column strength. For HEA700 profile, $W_s = 0.00624 \text{ m}^3$.

After substituting all input data and transforming the expressions into functions of the random variable vector, the limit state function can be formulated as follows:

$$G_3(\mathbf{X}) = 1 - \frac{|-35.44X_1 - 35.44X_4|}{0.00624X_7} \tag{15}$$

The Hasofer-Lind reliability index was determined using the FORM method, and the Monte Carlo method was used solely to verify the accuracy of the results. The reliability index values for the steel industrial building under different correlation matrix scenarios are presented in Table 6.

Table 6. Calculated values of the reliability index (β)

Probabilistic method	Case of correlation matrix			
	Case A ($\rho = 0$)	Case B ($\rho = 0.3$)	Case C ($\rho = 0.5$)	Case D ($\rho = 0.9$)
FORM	5.82	6.86	7.98	11.65
Monte Carlo	5.80	6.85	8.01	11.68

4. DISCUSSION

The analyses of all structures indicate that an increase in the correlation coefficient is associated with a corresponding rise in the reliability index. Across all investigated structural examples, the magnitude of this increase is generally consistent. Specifically, for a correlation coefficient of $\rho = 0.3$, the reliability index β increases by approximately 10%; for $\rho = 0.5$, the increase ranges from 30% to 40%; and for $\rho = 0.9$, the increase reaches approximately 100%. The precise percentage differences in the reliability index across structural scenarios are summarized in Table 7.

Table 7. Percentage increase in the reliability index (β) by correlation coefficient level, relative to Case A (no correlation)

Example	Case of correlation matrix		
	Case B ($\rho = 0.3$)	Case C ($\rho = 0.5$)	Case D ($\rho = 0.9$)
1 – „steel beam”	11%	37%	104%
2 – „steel arch”	10%	32%	110%
3 – „steel industrial building”	10%	37%	101%

Another key aspect of the study was to investigate how sensitive the reliability index is to variations in the correlation coefficient. The analysis revealed similar sensitivity values for the reliability index across the four cases (Case A–Case D) of the correlation matrix. Consequently, only two representative graphical results of the sensitivity analysis are presented for each example: Case A ($\rho = 0$) and Case D ($\rho = 0.9$). The corresponding results are illustrated in Figures 4-6.

In the FORM method, the α -vector indicates both the magnitude and the direction of each variable’s influence on the reliability index β . The absolute values of the components of the α -vector provide a quantitative measure of the influence of each corresponding variable on the reliability index β . Specifically, variables associated with larger α components exert a greater impact on the reliability index β , whereas those with smaller components contribute less significantly. The sign of each element determines the direction of influence: a positive α increases the probability of failure, reducing the reliability index β , whereas a negative α decreases the probability of failure, increasing β .

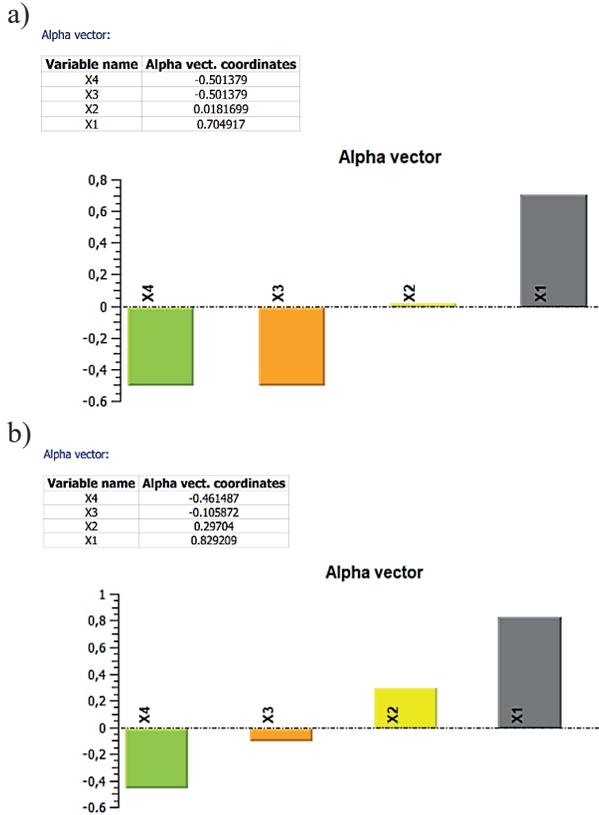


Fig. 4. Sensitivity of the reliability index to random variables for a steel beam: a) $\rho = 0$, b) $\rho = 0.9$

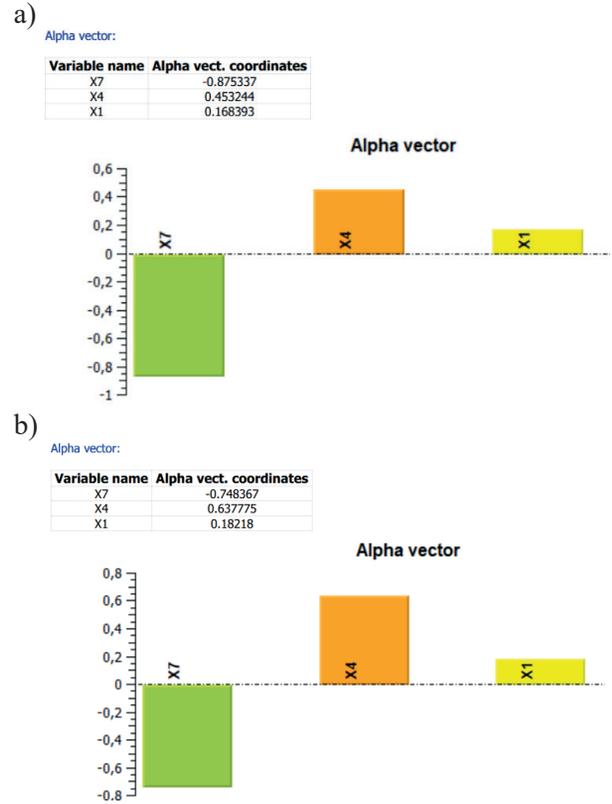


Fig. 6. Sensitivity of the reliability index to random variables for a steel industrial building: a) $\rho = 0$, b) $\rho = 0.9$

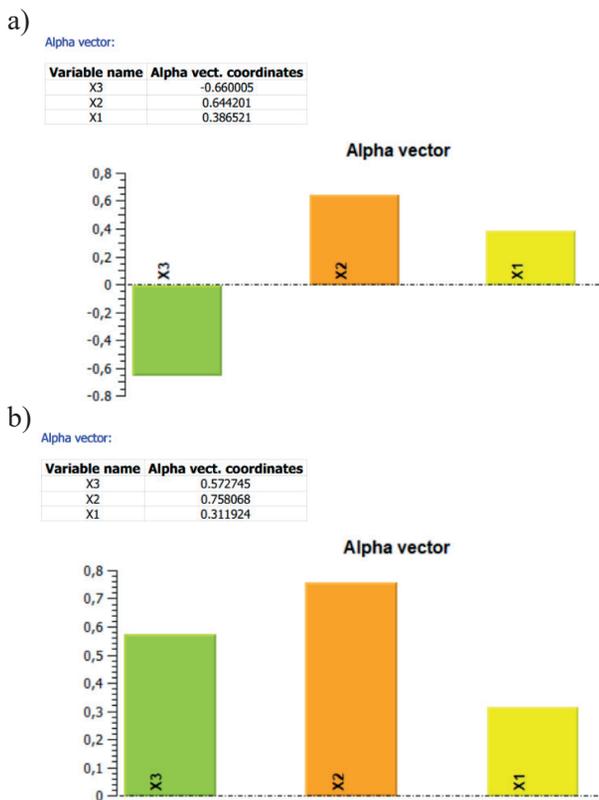


Fig. 5. Sensitivity of the reliability index to random variables for a steel arch: a) $\rho = 0$, b) $\rho = 0.9$

The analysis of the results for Example 2 and Example 3 (Figures 5 and 6) indicates that the sensitivity of the reliability index is generally similar across all random variables, regardless of whether they are correlated. In Example 2, the variable X_2 (variable load) exhibits the most significant influence, whereas X_1 (permanent load) has the least impact. Likewise, in Example 3, X_7 (yield point of S235JR steel) exerts the most decisive influence on the reliability index, while X_1 (roof self-weight per meter) has the lowest impact. An interesting phenomenon was observed in Example 2 (Fig. 5). The change in the sign of the α -vector component for variable X_3 from negative to positive indicates that X_3 , which previously decreased the probability of failure and increased the reliability index β , now increases the probability of failure, thereby reducing the reliability index β after accounting for correlations. The situation differs in Example 1. For $\rho = 0$, the highest impact on the reliability index is attributed to X_1 (uniform loading), with X_2 (beam length) exerting the least influence. For $\rho = 0.9$, the variable with the lowest impact shifts to X_3 (section modulus of the beam).

It can be concluded that the magnitude of the correlation coefficient significantly influences the sensitivity analysis results.

5. CONCLUSIONS

The analysis of the obtained results confirms that the FORM method provides sufficient accuracy compared with the Monte Carlo method, which is commonly regarded as a reference approach in structural reliability analysis. This observation aligns with conclusions from previous studies, which show that FORM provides reliable estimates of the reliability index β with significantly lower computational effort.

The results clearly indicate that the correlation coefficient between random variables plays a crucial role in the evaluation of structural safety and should not be neglected in reliability analyses. In particular, an increase in the correlation coefficient generally leads to higher values of the reliability index β , in line with trends observed in earlier investigations. Previous research has shown that both the magnitude and the sign of correlation significantly affect the estimated reliability, depending on the characteristics of the limit state function and the interaction between variables.

The study further confirms that proper consideration of correlations between random variables is essential for realistic reliability assessment. Similar conclusions have been reported in the literature, where neglecting correlations can lead to either underestimating or overestimating structural safety. Therefore, developing reliable tools and methodologies for

accurately estimating correlation coefficients remains an important research challenge.

An essential aspect of the present study is the sensitivity analysis of the reliability index. The results show that the correlation coefficient directly influences the sensitivity of the reliability index with respect to individual random variables. This observation aligns with previously published findings, which emphasize that correlations affect not only the reliability index itself but also its sensitivity measures. If the sensitivity of the reliability index to a particular random variable X_i is relatively low compared with other variables, that variable may be treated as deterministic in subsequent analyses. Such an approach reduces the number of random variables included in the mathematical model, leading to a significant decrease in computational time without a noticeable loss of accuracy. This strategy has also been recommended in earlier studies as an effective way to improve computational efficiency in reliability analysis.

In summary, the results show that accurately determining correlation coefficients is crucial for accurately describing the behavior and safety of the analyzed structures. Sensitivity analysis is a powerful tool for identifying the most influential random variables and supporting rational model simplification. Together, these aspects contribute to more efficient and reliable structural safety assessments.

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